Reconciling G_A/G_V, $<r_n^2>$ and $\mu_{p,n}$ in χQM with One Gluon Generated Configuration Mixing.

Harleen Dahiya and Manmohan Gupta

Department of Physics,
Centre of Advanced Study in Physics,
Panjab University,Chandigarh-160 014, India.

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Abstract

The spin polarization functions (Δu, Δd, Δs) for proton are calculated in the chiral quark model (χQM) with SU(3) symmetry breaking as well as configuration mixing generated by one gluon exchange forces for the NMC and the most recent E866 data. Besides reproducing the spin polarization functions Δu, Δd, Δs as well as G_A/G_V, it can accomodate nucleon magnetic moments and neutron charge radius as well, thus resolving the compatibility problem of these parameters which could not be achieved in constituent quark models.

The Constituent Quark Model (CQM), despite its impressive performance in explaining low energy hadronic matrix elements, is unable to account for the EMC effect or the "proton spin crisis" [1] which indicated that only a small portion of the proton spin is carried by the valence quarks, as well as the presence of significant negative strange quark polarization in the proton quark sea [2]. CQM is also unable to explain the results of the NMC [3] and E866 [4] experiments which have shown that the Gottfried sum rule [5] is violated, indicating that the $\bar{d}$ density is larger than $\bar{u}$ density in the nucleon sea. Apart from the problems faced by CQM in explaining the spin content of nucleons, it is also saddled with another problem, *viz.*, it has been shown [6] that it is not possible to have a simultaneous reconciliation of G_A/G_V, charge form factors of nucleon and magnetic moments of nucleons ($\mu_{p,n}$) in the CQM. This problem becomes more acute if one considers neutron charge radius ($<r_n^2>$).

The EMC effect and related issues have been successfully addressed in the chiral quark model (χQM) [7, 8, 9, 10], originally conceived by Weinberg [11],

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subsequently developed by Manohar and Georgi [12]. In particular, the χQM is able to explain not only the spin content of the nucleon but is also able to give a fair description of flavor content of the nucleons, violation of Gottfried sum rule, strange quark content in the nucleon, G_A/G_V as well as the magnetic moments of the baryons etc. [7, 8, 9, 10].

Recently it has been claimed that, within the χQM, neutron charge radius can also be reproduced through the Foldy term [13], however very recently it has been shown by Isgur [14] that the Foldy term gets cancelled against a contribution to the Dirac form factor F_1 to leave intact the interpretation of neutron electric charge form factor G_E^n as arising from the neutron's rest frame charge distribution. Therefore, in the light of the work of Isgur, it becomes essential that $< r_n^2 >$ should be reproduced by the form factor F_1. Thus, even in the χQM, the question of compatibility of G_A/G_V, $< r_n^2 >$ and $\mu_{p,n}$ remains.

In the context of CQM, it is well known that there are several low energy parameters which can be explained by including one gluon mediated configuration mixing [15, 16, 17, 18, 19]. It has also been shown that one gluon mediated configuration mixing is able to generate $< r_n^2 >$ as well as it improves the fit of magnetic moment of baryons [17, 18, 19]. In view of the fact that the χQM incorporates the basic features of constituent quark model, therefore it becomes interesting to examine the implications of configuration mixing in χQM (χQM$_{gcm}$). The purpose of the present communication on the one hand is to examine, within the χQM, the implications of one gluon mediated configuration mixing for spin polarizations and related issues while on the other hand it is to investigate the issue of compatibility of G_A/G_V, neutron charge radius and $\mu_{p,n}$.

To understand the implications of one gluon mediated configuration mixing for G_A/G_V, we first calculate the quark spin polarizations in the χQM. The basic process, in the χQM, is the emission of a Goldstone Boson which further splits into $q\bar{q}$ pair, for example,

$$q_\pm \to GB^0 + q'_\mp \to (q\bar{q'}) + q'_\mp. \tag{1}$$

The above process can be expressed through the Lagrangian

$$\mathcal{L} = g_8 \bar{q} \Phi q, \tag{2}$$

where g_8 is the coupling constant,

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \tag{3}$$

and

$$\Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \beta\frac{\eta}{\sqrt{6}} + \zeta\frac{\eta'}{\sqrt{3}} & \pi^+ & \alpha K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \beta\frac{\eta}{\sqrt{6}} + \zeta\frac{\eta'}{\sqrt{3}} & \alpha K^0 \\ \alpha K^- & \alpha \bar{K}^0 & -\beta\frac{2\eta}{\sqrt{6}} + \zeta\frac{\eta'}{\sqrt{3}} \end{pmatrix}. \qquad (4)$$

SU(3) symmetry breaking is introduced by considering different quark masses $m_s > m_{u,d}$ as well as by considering the masses of non-degenerate Goldstone Bosons $M_{K,\eta} > M_\pi$ [9, 10, 20] , whereas the axial U(1) breaking is introduced by $M_{\eta'} > M_{K,\eta}$ [7, 9, 10, 20]. The parameter a($=|g_8|^2$) denotes the transition probability of chiral fluctuation or the splittings $u(d) \rightarrow d(u)+\pi^{+(-)}$, whereas $\alpha^2 a$ denotes the probability of transition $u(d) \rightarrow s + K^{-(0)}$. Similarly $\beta^2 a$ and $\zeta^2 a$ denote the probability of $u(d,s) \rightarrow u(d,s) + \eta$ and $u(d,s) \rightarrow u(d,s) + \eta'$ respectively.

The one gluon exchange forces [15] generate the mixing of the octet in $(56,0^+)_{N=0}$ with the corresponding octets in $(56,0^+)_{N=2}$, $(70,0^+)_{N=2}$ and $(70,2^+)_{N=2}$ harmonic oscillator bands [16]. The corresponding wave function of the nucleon is given by

$$|B> = (|56,0^+>_{N=0} \, cos\theta + |56,0^+>_{N=2} \, sin\theta)cos\phi$$

$$+ (|70,0^+>_{N=2} \, cos\theta + |70,2^+>_{N=2} \, sin\theta)sin\phi. \qquad (5)$$

In the above equation it should be noted that $(56,0^+)_{N=2}$ does not affect the spin-isospin structure of $(56,0^+)_{N=0}$, therefore the mixed nucleon wave function can be expressed in terms of $(56,0^+)_{N=0}$ and $(70,0^+)_{N=2}$, which we term as non trivial mixing [19] and is given as

$$\left|8,\frac{1}{2}^+\right> = cos\phi|56,0^+>_{N=0} +sin\phi|70,0^+>_{N=2}, \qquad (6)$$

where

$$|56,0^+>_{N=0,2} = \frac{1}{\sqrt{2}}(\chi'\phi' + \chi''\phi'')\psi^s, \qquad (7)$$

$$|70,0^+>_{N=2} = \frac{1}{2}[(\psi''\chi' + \psi'\chi'')\phi' + (\psi'\chi' - \psi''\chi'')\phi'']. \qquad (8)$$

The spin and isospin wave functions, χ and ϕ, are given below

$$\chi' = \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow), \quad \chi'' = \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow),$$

$$\phi'_p = \frac{1}{\sqrt{2}}(udu - duu), \quad \phi''_p = \frac{1}{\sqrt{6}}(2uud - udu - duu),$$

$$\phi'_n = \frac{1}{\sqrt{2}}(udd - dud), \quad \phi''_n = \frac{1}{\sqrt{6}}(udd + dud - 2ddu).$$

For the definition of the spatial part of the wave function, (ψ^s, ψ', ψ'') as well as the definitions of the spatial overlap integrals we refer the reader to references [17] and [19].

The contribution to the proton spin, defined through the equation

$$\Delta q = q_\uparrow - q_\downarrow + \bar{q}_\uparrow - \bar{q}_\downarrow, \tag{9}$$

using Equation(4) and following Linde *etal.* [9], can be expressed as

$$\Delta u = cos^2\phi \left[\frac{4}{3} - \frac{a}{3}(7 + 4\alpha^2 + \frac{4}{3}\beta^2 + \frac{8}{3}\zeta^2)\right] + sin^2\phi \left[\frac{2}{3} - \frac{a}{3}(5 + 2\alpha^2 + \frac{2}{3}\beta^2 + \frac{4}{3}\zeta^2)\right], \tag{10}$$

$$\Delta d = cos^2\phi \left[-\frac{1}{3} - \frac{a}{3}(2 - \alpha^2 - \frac{1}{3}\beta^2 - \frac{2}{3}\zeta^2)\right] + sin^2\phi \left[\frac{1}{3} - \frac{a}{3}(4 + \alpha^2 + \frac{1}{3}\beta^2 + \frac{2}{3}\zeta^2)\right], \tag{11}$$

and

$$\Delta s = -a\alpha^2. \tag{12}$$

Before one presents and discusses the results pertaining to Equations (10), (11) and (12), for a better appreciation of the role of configuration mixing and symmetry breaking we have considered the case of SU(3) symmetry as well. The SU(3) symmetric calculations can easily be obtained from the equations (10), (11) and (12) by considering $\alpha, \beta = 1$. The corresponding equations can be expressed as

$$\Delta u = cos^2\phi \left[\frac{4}{3} - \frac{a}{9}(37 + 8\zeta^2)\right] + sin^2\phi \left[\frac{2}{3} - \frac{a}{9}(23 + 4\zeta^2)\right], \tag{13}$$

$$\Delta d = cos^2\phi \left[-\frac{1}{3} - \frac{2a}{9}(\zeta^2 - 1)\right] + sin^2\phi \left[\frac{1}{3} - \frac{a}{9}(16 + 2\zeta^2)\right], \tag{14}$$

and

$$\Delta s = -a. \tag{15}$$

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Para-meter	Expt value	Without configuration mixing					ϕ	With configuration mixing				
		CQM	χQM with SU(3) symmetry		χQM with SU(3) symmetry breaking			CQM_{gcm}	χQM_{gcm} with SU(3) symmetry		χQM_{gcm} with SU(3) symmetry breaking	
			NMC	E866	NMC	E866			NMC	E866	NMC	E866
Δu	0.85 [21]	1.33	.79	.81	.96	.99	20^0	1.26	.74	.76	.90, .86*	.92, .88*
							18^0	1.27	.75	.77	.91, .87*	.93, .89*
							16^0	1.28	.76	.78	.92, .88*	.94, .90*
							14^0	1.29	.77	.79	.93, .89*	.95, .91*
Δd	-.41 [21]	-.33	-.35	-.37	-.40	-.41	20^0	-.26	-.30	-.31	-.32, -.36*	-.34, -.38*
							18^0	-.27	-.31	-.32	-.33, -.37*	-.35, -.39*
							16^0	-.28	-.32	-.33	-.34, -.38*	-.36. -.40*
							14^0	-.29	-.33	-.34	-.35, -.39*	-.37, -.41*
Δs	-.07 [21]	0	-.1	-.12	-.02	-.02		0	-.1	-.12	-.02, -.06*	-.02, -.06*
G_A/G_V	1.267 [22]	1.66	1.14	1.18	1.35	1.40	20^0	1.52	1.04	1.07	1.22, 1.22*	1.26, 1.26*
							18^0	1.54	1.06	1.09	1.24, 1.24*	1.28, 1.28*
							16^0	1.56	1.08	1.11	1.26, 1.26*	1.30, 1.30*
							14^0	1.58	1.10	1.13	1.28, 1.28*	1.32, 1.32*

* Values after inclusion of the contribution from anomaly [23].

Table 1: The calculated values of spin polarization functions and G_A/G_V.

In Table 1, we have presented the results of our calculations. First of all, for χQM with SU(3) symmetry breaking as well as configuration mixing, we have carried out a χ^2 fit to Δu, Δd, Δs, G_A/G_V and other related parameters (details of which will be published elsewhere), however, in the fit we have taken $\phi \simeq 20^0$, a value dictated by consideration of neutron charge radius [17]. In the table we have also considered a few more values of the mixing parameter ϕ in order to study the variation of spin distribution functions with ϕ. The parameter a is taken to be 0.1, as considered by other authors [7, 8, 9, 10]. The symmetry breaking parameters obtained from χ^2 fit are $\alpha = .4$ and $\beta = .7$ for the data corresponding to recent E866 [4] as well as NMC [3] data . The parameter ζ is constrained by the expressions $\zeta = -0.7 - \beta/2$ and $\zeta = -\beta/2$ for the NMC and E866 experiments respectively, which essentially represent the fitting of deviation from Gottfried sum rule [5]. Further, while presenting the results of SU(3) symmetry breaking case without configuration mixing $(\phi = 0^0)$, we have used the same values of parameters α and β, primarily to understand the role of configuration mixing for this case. The SU(3) symmetry calculations based on Equations (13), (14) and (15) are obtained by taking $\alpha = \beta = 1, \phi = 20^0$ and $\alpha = \beta = 1, \phi = 0^0$ respectively for with and without configuration mixing. For the sake of completion, we have also presented the results of CQM with and without configuration mixing. In this case the spin polarization functions can easily be found from equations (10), (11) and (12), for example,

$$\Delta u = cos^2\phi[\frac{4}{3}] + sin^2\phi[\frac{2}{3}], \tag{16}$$

$$\Delta d = cos^2\phi[-\frac{1}{3}] + sin^2\phi[\frac{1}{3}], \tag{17}$$

and

$$\Delta s = 0. \tag{18}$$

From Table 1, it is clear that χQM with SU(3) symmetry breaking along with configuration mixing generated by one gluon exchange forces is able to give an excellent fit to the spin polarization data for symmetry breaking parameters $\alpha = .4$ and $\beta = .7$. In order to appreciate the role of configuration mixing in affecting the fit, we first compare the results of CQM with those of CQM$_{gcm}$. One observes that configuration mixing corrects the result of the quantities in the right direction but this is not to the desirable level. Further, in order to understand the role of configuration mixing and SU(3) symmetry with and without breaking in χQM, we can compare the results of χQM with SU(3) symmetry to those of χQM$_{gcm}$ with SU(3) symmetry. Curiously χQM$_{gcm}$ compares unfavourably with χQM in case of the calculated quantities. This indicates that configuration mixing alone is not enough to generate an appropriate fit in χQM. However when χQM$_{gcm}$ is used with SU(3) symmetry breaking then the results show uniform improvement over the corresponding results of χQM with SU(3) symmetry breaking. It is interesting to note that the values of α and β are in agreement with other recent calculations [7, 9, 10].

After having seen that χQM$_{gcm}$ is able to accomodate G_A/G_V, one turns to the question of compatibility of G_A/G_V, $\mu_{p,n}$ and $< r_n^2 >$. In this regard, we first evaluate magnetic moments in the χQM$_{gcm}$. In this context, it has been shown recently by Cheng and Li [20] that χQM, incorporating the symmetry breaking effects, leads to formula which is similar to CQM, for example,

$$\mu(B) = (1 + \kappa_{spin} + \kappa_{orbit})\mu(B)_v, \tag{19}$$

where $(1 + \kappa_{spin} + \kappa_{orbit})$ is the Cheng and Li scale factor and $\mu(B)_v$ is the magnetic moment in the CQM. The Cheng and Li scale factor can be absorbed in the quark masses, thus magnetic moments calculated in χQM essentially reduce to using appropriate 'quark mass scales' for fitting the magnetic moments.

In Table 2, we have presented the magnetic moments with the wavefunctions expressed through equation (6), in terms of μ_{56} and μ_{70},

$$\mu(N) = cos^2\phi < 56|M|56 > + sin^2\phi < 70|M|70 >$$

$$= cos^2\phi(\mu_{56}) + sin^2\phi(\mu_{70}),$$

where M is the magnetic moment operator. It is evident from the table that for a good range of ϕ we can fit the magnetic moments of the nucleons. As has

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	ϕ	$\mu(p)$	$\mu(n)$	R^2 (GeV^{-2})	$<r_n^2>$ (GeV^{-2})
μ_{56}	-	$\frac{1}{3}(4\mu_u - \mu_d)$	$\frac{1}{3}(4\mu_d - \mu_u)$	-	-
μ_{70}		$\frac{1}{3}(2\mu_u+\mu_d)$	$\frac{1}{3}(2\mu_d + \mu_u)$	-	-
Expt value	-	2.793	-1.913	-	2.82
Calculated values	-20^0	2.766	-1.767	8	2.64
				9	2.93
				10	3.23
	-18^0	2.804	-1.805	8	2.39
				9	2.65
				10	2.91
	-16^0	2.841	-1.843	10	2.61
				11	2.84
				12	3.07
	-14^0	2.882	-1.882	12	2.71
				13	2.91
				14	3.11

Table 2: The magnetic moments and neutron charge radius for different values of ϕ and R^2. The 'effective masses' for the constituent quarks used here are $m_u = m_d = 0.313 \ GeV, \ m_s = 0.513 \ GeV$.

been shown [19], this not only reproduces nucleon magnetic moments but is also able to give a good fit to other magnetic moments . Further, the magnetic moments are independent of the sign of mixing angle ϕ.

After having realized that the χQM_{gcm} can explain G_A/G_V and the nuclear magnetic moments, we have tried to investigate whether it is able to reproduce the neutron charge radius $<r_n^2>$. As it has been emphasized earlier that the Foldy term, reproducible within χQM, gets cancelled against a contribution to the Dirac form factor [14], therefore to calculate the neutron charge radius we have effectively replaced $G_E^n(q^2)_{\chi QM} \Rightarrow G_E^n(q^2)_{QM}$. In the CQM similar calculations have been done [6]. In order to emphasize its dependence on mixing angle ϕ we reproduce here some of the essential details [17].

The neutron charge radius is usually expressed in terms of the slope of the electric form factor $G_E^n(q^2)$, the experimental value [24] of which is given as

$$(\frac{dG_E^n(q^2)}{d|q^2|})_{q^2=0} = 0.47 \pm 0.01 \ GeV^{-2}. \tag{20}$$

If we assign the nucleon to a pure 56 (with the spin expressed in terms of

Pauli spinors), the neutron electric form factor vanishes for all q^2. Considering our complete wavefunction with the $56 - 70$ mixing the neutron charge radius can be expressed as

$$< r_n^2 >= 6(\frac{dG_E^n(q^2)}{d|q^2|})_{q^2=0} = \sqrt{\frac{2}{3}}R^2(-tan\phi), \tag{21}$$

where ϕ is the mixing angle which is negative and R^2 is the shape factor [6, 17] for the harmonic oscillator wave function. We have given the calculated values of neutron charge radius $< r_n^2 >$ $(= 6b)$ as function of ϕ and R^2 in Table 2. From the table one finds that one is able to reproduce fairly well the experimental value of $< r_n^2 >$, $viz.$, 2.82 GeV^{-2} in the χQM$_{gcm}$ by considering the same values of parameter ϕ which reproduced the values of G_A/G_V and nuclear magnetic moments.

In conclusion, we would like to mention that we have calculated the spin polarization functions (Δu, Δd and Δs) in the χQM with SU(3) symmetry breaking as well as one gluon generated configuration mixing for the NMC and the most recent E866 data. Besides reproducing G_A/G_V and magnetic moments, it can also accomodate the neutron charge radius, thus resolving the compatibility problem of these parameters which could neither be achieved in the CQM with configuration mixing nor in the χQM with and without SU(3) symmetry breaking.

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